MATHEMATICS

Chapter 9: SEQUENCE AND SERIES

SEQUENCE AND SERIES

Top Definitions

1. A Sequence is an ordered list of numbers and has the same meaning as conversational english. Asequence is denoted by $<a_n>_{n\geq 1} = a_1, a_2, a_3, \ldots\ldots.a_n$.

2. The various numbers occurring in a sequence are called its terms.

3. A sequence containing finite number of terms is called a finite sequence. A finite sequence has the last term.

4. A sequence which is not a finite sequence, i.e., containing an infinite number of terms is called aninfinite sequence. There is no last term in an infinite sequence.

5. A sequence is said to be an arithmetic progression if every term differs from the preceding term by a constant number. For example, the sequence $a_1, a_2, a_3, \ldots a_n$ is called an arithmetic sequence or AP if $a_{n+1} = a_n + d$.

   For all $n \in N$, where d is a constant called the common difference of AP.

6. A is the arithmetic mean of two numbers a and b if a, A and b forms AP.

7. A sequence is said to be a geometric progression or GP if the ratio of any tem to its preceding term isthe same throughout. Constant ratio is a common ratio denoted by r.

8. If three numbers are in GP, then the middle term is called the geometric mean of the other two.

Top Concepts

1. A sequence has a definite first member, second member, third member and so on.

2. The n^{th} term $<a_n>$ is called the general term of the sequence

3. Fibonacci sequence 1, 1, 2, 3, 5, 8, .. is generated by a recurrence relation given by

   $a_1 = a_2 = 1.$

   $a_3 = a_1 + a_2 \ldots\ldots$

   $a_n = a_{n-2} + a_{n-1}, n > 2.$

4. A sequence is a function with a domain, the set of natural numbers or any of its subsets

of the type {1, 2, 3, ... k}.

5. If the number of terms is three with a common difference 'd', then the terms are $a - d,\ d$ and $a + d$
6. If the number of terms is four with common difference '2d', then the terms are $a - 3d,\ \ a - d,\ a + d$ and $a + 3d$.
7. If the number of terms is five with a common difference 'd', then the terms are $a - 2d,\ a - d,\ a,\ a + d$ and $a + 2d$.
8. If the number of terms is six with a common difference '2d', then the terms are $a - 5d, a - 3d, a - d, a + d, a + 3d$ and $a + 5d$.
9. The sum of the series is the number obtained by adding the terms.
10. The general form of AP is a, a + d, a + 2d, ...a + (n - 1) d. The term a is called the **first term** of AP and d is called the **common difference** of AP. The term d can be any real number.
11. If d>0, then AP is increasing, if d < 0 then AP is decreasing, and if d = 0, then AP is constant.
12. For AP, a, (a + d), (a + 2d), ..., (λ − 2d), (λ − d), λ with the first term a and common difference d and last term λ and general term is λ − (n − 1)d.
13. Let a be the first term and d be the common difference of AP with 'm' terms. Then nth term from theend is the $(m - n + 1)^{th}$ term from the beginning.
14. Properties of AP
    i. If a constant is added to each term of AP, then the resulting sequence is also AP.
    ii. If a constant is subtracted from each term of AP, then the resulting sequence is also AP.
    iii. If each term of AP is multiplied by a constant, then the resulting sequence is also an AP.
    iv. If each term of AP is divided by a non-zero constant, then the resulting sequence is also AP.
15. The arithmetic mean A of any two numbers a and b is given by $\frac{a+b}{2}$.
16. General form of GP: a, ar, ar^2, ar^3,....., where a is the first term and r is the constant ratio. The term r can take any non-zero real number.
17. A sequence in GP will remain in GP if each of its terms is multiplied by a non-zero constant.

18. A sequence obtained by multiplying the two GPs term by term results in GP with common ratio theproduct of the common ratio of the two GPs.

19. The reciprocals of the terms of a given GP form a GP with common ratio $\frac{1}{r}$.

20. If each term of GP is raised to the same power, then the resulting sequence also forms GP.

21. The geometric mean (GM) of any two positive numbers a and b is given by $\sqrt{ab}$.

22. Let A and G be AM and GM of two given positive real numbers a and b, respectively, then A ≥ G.

    Where $A = \frac{a+b}{2}$ and G = $\sqrt{ab}$

23. Let A and G be AM and GM of two given positive real numbers a and b, respectively. Then, the quadratic equation having a and b as its roots is x_2 - 2Ax + G = 0.

24. Let A and G be AM and GM of two given positive real numbers a and b, respectively. Then the given numbers are $A \pm \sqrt{A^2 - G}$.

25. If AM and GM are in the ratio m:n, then the given numbers are in the ratio

    $m + \sqrt{m^2 - n^2} : m - \sqrt{m^2 - n^2}$.

Top Formulae

1. The n[th] term or general term of AP is $a_n = a + (n - 1)d$, where a is the first term and d is the commondifference.

2. The general term of AP given its last term is $l - (n - 1)d$.

3. A sequence is AP if and only if its nth term is a linear expression in n,
   $A_n = Xn + Y$, where X and Y are constants.

4. Let a, a + d, a + 2d,.., a + (n − 1)d be AP. Then
   $S_n = \frac{n}{2}[2a + (n-1)d]$ or $S_n = \frac{n}{2}[a + \ell]$ where $\lambda = a + (n - 1)d$

5. A sequence is AP if and only if the sum of its nth term is an expression of the form X_n^2 + Y, where X and Y are constants.

6. If the terms of an AP are selected in regular intervals, then the selected terms form an AP.

   Let a_n, a_{n+1} and a_{n+2} be the consecutive terms of AP.

Then $2a_{n+1} + = a_n\ a_{n+2}$.

7. Let $A_1, A_2, A_3, \ldots A_n$ be n numbers between a and b such that a, $A_1, A_2, A_3, \ldots A_n$, b is AP. The n numbers between a and b are as follows

$$A_1 = a + d = a + \frac{b-a}{n+1}$$

$$A_2 = a + 2d = a + \frac{2(b-a)}{n+1}$$

$$A_3 = a + 3d = a + \frac{3(b-a)}{n+1}$$

$$\ldots \quad \ldots \quad \ldots \quad \ldots$$

$$\ldots \quad \ldots \quad \ldots \quad \ldots$$

$$A_n = a + nd = a + \frac{n(b-a)}{n+1}$$

8. General term of GP is ar^{n-1}, where a is the first term and r is the common ratio.

9. If the number of terms is 3 with the common ratio r, then the selection of terms should be $\frac{a}{r}$, and ar.

10. If the number of terms is 4 with the common ratio r2 , then the selection of terms should be $\frac{a}{r^3}$, a ar and ar^2.

11. If the number of terms is 5 with the common ratio r, then the selection of terms should be $\frac{a}{r^2}$, a, a, ar and ar^2.

12. Sum to the first n terms of GP $S_n = a + ar + ar^2 + \ldots + ar^{n-1}$

(i) If r = 1, $S_n = a + a + a + \ldots + a$ (n terms) = na.

(ii) If r<1, $S_n = \frac{q(1-r^n)}{1-r}$.

(iii) If r>1, $S_n = \frac{a(r^n-1)}{r-1}$.

13. Let $G_1, G_2, \ldots\ldots, G_n$ be n numbers between positive numbers a and b such that a, G_1, $G_2, G_3, \ldots\ldots G_n$, b is GP.

Thus $b = br^{n+1}$, or $r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

$G_1 = ar = a\left(\frac{b}{a}\right)^{\frac{1}{n+1}}$, $G_2 = ar^2 = a\left(\frac{b}{a}\right)^{\frac{2}{n+1}}$, $G_3\ ar^3 = a\left(\frac{b}{a}\right)^{\frac{3}{n+1}}$

$G_n = ar^n = a\left(\frac{b}{a}\right)^{\frac{n}{n+1}}$

14. The sum of infinite GP with the first term a and common ratio r ($-1 < r < 1$) is $S = \frac{a}{1-r}$.

Some Special Series

1. The sum of first n natural numbers is

$$1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$$

2. Sum of squares of the first n natural numbers

$$1^2+2^2+3^2+\ldots\ldots n^2 = \frac{n(n+1)(2n+1)}{6}$$

3. Sum of cubes of first n natural numbers

$$1^3 + 2^3 + \ldots + n^3 = \frac{n^2(n+1)^2}{4} = \left[n(n+1)\right]^2$$

4. Sum of powers of 4 of first n natural numbers

$$1^4 + 2^4 + 3^4 + \ldots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

5. Consider the series $a_1 + a_2 + a_3 + a_4 + \ldots + an + \ldots$.

   If the differences in $a_2 - a_1$, $a_3 - a_2$, $a_4 - a_3$, …. are in AP, then the nth term is given by $a_n = an^2 + bn + c$, where a, b and c are constants.

6. Consider the series $a_1 + a_2 + a_3 + a_4 + \ldots + an + \ldots$.

   If the differences in $a_2 - a_1$, $a_3 - a_2$, $a_4 - a_3$, …. are in GP then the nth term is given by

   $an = ar^{n-1} + b + c$, where a, b and c are constants.

MIND MAP : LEARNING MADE SIMPLE CHAPTER - 9

Sequences and Series

Definitions

Sequence is a function whose domain is the set of N natural numbers.

Real sequence: A sequence whose range is a subset of R is called a real sequence

Series: If $a_1, a_2, a_3, \ldots a_n$, is a sequence, then the expression $a_1+a_2+a_3+\ldots a_n+\ldots$ is a series. A series is finite or infinite according to the no. of terms in the corresponding sequence as finite or infinite.

Progression: Those sequences whose terms follow certain patterns are called progression.

Arithmetic Progression (A.P)

An A.P. is a sequence in which terms increase or decrease regularly by the fixed number (same constant). This fixed number is called 'common difference' of the A.P. If 'a' is the first term & 'd' is the common difference and 'l' is the last term of A.P., then general term or the nth term of the A.P. is given by $a_n = a + (n-1)d$.

The sum S_n of the first n terms of an A.P. is given by

$$S_n = \frac{n}{2}[2a+(n-1)d] = \frac{n}{2}[a+l]$$

Arithmetic Mean (AM)

If three numbers are in A.P., then the middle term is called AM between the other two, so if a, b, c, are in A.P., b is AM of a and c.

AM for any 'n' +ve numbers $a_1, a_2, a_3, \ldots, a_n$ is

$$AM = \frac{a_1+a_2+a_3+\ldots+a_n}{n}$$

n-Arithmetic Mean between Two Numbers: If a, b are any two given numbers & a, $A_1, A_2, \ldots, A_n$, b are in A.P. then $A_1, A_2, \ldots, A_n$ are n AM's between a & b.

$$A_1 = a+\frac{b-a}{n+1}, A_2 = a+\frac{2(b-a)}{n+1} \ldots A_n = a+\frac{n(b-a)}{n+1}$$

or $A_1 = a+d, A_2 = a+2d, \ldots, A_n = a+nd$, where $d = \frac{b-a}{n+1}$

Sum upto terms of some Special Sequences

- Sum of first 'n' natural numbers
$$\sum_{k=1}^{n} k = 1+2+3+\ldots+n = \frac{n(n+1)}{2}$$
- Sum of squares of first n natural numbers
$$\sum_{k=1}^{n} k^2 = 1^2+2^2+3^2+\ldots+n^2 = \frac{n(n+1)(2n+1)}{6}$$
- Sum of cubes of first n natural numbers
$$\sum_{k=1}^{n} k^3 = 1^3+2^3+3^3+\ldots+n^3 = \left(\frac{n(n+1)}{2}\right)^2 = \left(\sum_{k=1}^{n} k\right)^2$$
- Sum of first 'n' odd natural numbers
$$\sum_{k=1}^{n}(2k-1) = 1+3+5+\ldots+(2n-1) = n^2$$

Geometric Progression (G.P.)

A sequence is said to be a geometric progression or G.P., if the ratio of any term to its preceding term is same throughout.This constant factor is called **'Common ratio'**. Usually we denote the first term of a G.P. by 'a' and its common ratio by 'r'. The gereral or nth term of G.P. is given by $a_n = ar^{n-1}$ The sum S_n of the first n terms of G.P. is given by

$$S_n = \frac{a(r^n-1)}{r-1} \text{ or } \frac{a(1-r^n)}{1-r} \text{ if } r \neq 1$$

Sum of infinite terms of G.P. is given by

$$S_n = \frac{a}{1-r}; |r|<1$$

Geometric Mean (GM)

- If a, b, c are in G.P., b is the GM between a & c, $b^2 = ac$, therefore $b = \sqrt{ac}$; a > 0, c > 0
- If a, b are two given numbers & a, $G_1, G_2, \ldots, G_n$, b are in G.P., then $G_1, G_2, G_3 \ldots G_n$ are n GMs between a & b.

$$G_1 = a\cdot\left(\frac{b}{a}\right)^{\frac{1}{n+1}} G_2 = a\cdot\left(\frac{b}{a}\right)^{\frac{2}{n+1}} \ldots\ldots G_n = a\cdot\left(\frac{b}{a}\right)^{\frac{n}{n+1}}$$

or

$$G_1 = ar,\ G_2 = ar^2,\ \ldots\ G_n = ar^{n-1}$$

where $r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$,

Arithmetico Geometric Series

In a series,where each term is formed by multiplying the corresponding term of an AP & GP is called Arithmetico-Geometric Series.

Eg: $1+3x+5x^2+7x^3+\ldots$

Here, 1, 3, 5, … are in AP and 1, x, x^2, … are in GP.

Properties of AM & GM between two quantities

- If A and G are respectively arithmetic and geometric means between two positive numbers a and b; then the quadratic equation have a, b as it's root is $x^2-2Ax+G^2=0$
- If A and G be the AM & GM between two positive numbers, then the numbers are $A \pm \sqrt{A^2-G^2}$

Relationship between AM and GM

Let A and G be the AM and GM of two given positive real numbers a & b, respectively. Then,

$$A = \frac{a+b}{2} \text{ and } G = \sqrt{ab}$$

Thus, we have , $A - G = \frac{a+b}{2} - \sqrt{ab} = \frac{a+b-2\sqrt{ab}}{2}$

$$= \frac{(\sqrt{a}-\sqrt{b})^2}{2} \geq 0 \quad \ldots\ldots\ldots\ldots (1)$$

From (1), we obtain the relationship $A \geq G$

Important Questions

Multiple Choice questions-

Question 1. Let Tr be the rth term of an A.P. whose first term is a and the common difference is d. If for some positive integers m, n, m ≠ n, T m = 1/n and T n = 1/m then (a-d) equals to

(a) 0

(b) 1

(c) 1/mn

(d) 1/m + 1/n

Question 2. The first term of a GP is 1. The sum of the third term and fifth term is 90. The common ratio of GP is

(a) 1

(b) 2

(c) 3

(d) 4

Question 3. If a is the first term and r is the common ratio then the nth term of GP is

(a) $(ar)^{n-1}$

(b) $a \times r^{n}$

(c) $a \times r^{n-1}$

(d) None of these

Question 4. The sum of odd integers from 1 to 2001 is

(a) 10201

(b) 102001

(c) 100201

(d) 1002001

Question 5. If a, b, c are in AP and x, y, z are in GP then the value of $x^{b-c} \times y^{c-a} \times z^{a-b}$ is

(a) 0

(b) 1

(c) -1

(d) None of these

Question 6. An example of geometric series is

(a) 9, 20, 21, 28

(b) 1, 2, 4, 8

(c) 1, 2, 3, 4

(d) 3, 5, 7, 9

Question 7. Three numbers from an increasing GP of the middle number is doubled, then the new numbers are in AP. The common ratio of the GP is

(a) 2

(b) $\sqrt{3}$

(c) $2 + \sqrt{3}$

(d) $2 - \sqrt{3}$

Question 8. An arithmetic sequence has its 5th term equal to 22 and its 15th term equal to 62. Then its 100th term is equal to

(a) 410

(b) 408

(c) 402

(d) 404

Question 9. Suppose a, b, c are in A.P. and a^2, b^2, c^2 are in G.P. If $a < b < c$ and $a + b + c = 3/2$, then the value of a is

(a) $1/2\sqrt{2}$

(b) $1/2\sqrt{3}$

(c) $1/2 - 1/\sqrt{3}$

(d) $1/2 - 1/\sqrt{2}$

Question 10. If the positive numbers a, b, c, d are in A.P., then abc, abd, acd, bcd are

(a) not in A.P. / G.P. / H. P.

(b) in A.P.

(c) in G.P.

(d) in H.P.

Short Questions:

1. Which term of the sequence 27,24,21,18, ………… is zero?

2. The last term of the series 8,4,0, ……………. is – 24. Find the total number of terms.

3. Seven times the 7th term of a series is equal to eleven times of its 11th term. Find the 18th term of the series.

4. Prove that the sum of $(m + n)^{th}$ term and $(m - n)^{th}$ terms of an A.P. is twice of its m^{th} term.

5. Insert three Arithmetic means between 3 and 19.

6. If – 8, A_1, A_2 are in Arithmetic progression (A.P.), then find the value of A_1, A_2.

7. A person pays first instalment of Rs. 100 towards his loan. If he increases his instalment every month by Rs. 5, then what will be his 30th instalment

8. Which term of the sequence $\sqrt{3}$, 3, $3\sqrt{3}$ ……. is 729?

9. How many terms are required in GP. $3, 3^2, 3^3$ …………, so that their sum would be 120?

10. If A.M. and GM. of roots of a quadratic equation are 8 and 5 respectively, then find the quadratic equation.

Long Questions:

1. 150 workers were engaged to finish a job in a certain no. of days 4 workers dropped out on the second day, 4 more workers dropped out on the third day and so on. It took 8 more days to finish the work find the no. of days in which the work was completed.

2. Prove that the sum to n terms of the series 11 + 103 +1005 + _ _ _ _ _ _ _ is $\frac{10}{9}(10^n - 1) + n^2$

3. The ratio of A M and G. M of two positive no. a and b is m : n show that

4. Between 1 and 31, m number have been inserted in such a way that the resulting sequence is an A.P. and the ratio of 7th and (m-1)th no. is 5:9 find the value of m.

5. The Sum of two no. is 6 times their geometric mean, show that no. are in the ratio $(3 + 3\sqrt{2}) : (3 - 2\sqrt{2})$

Assertion Reason Questions:

1. In each of the following questions, a statement of Assertion is given followed by a corresponding statement of Reason just below it. Of the statements, mark the correct answer as.

   Assertion (A) : If the sequence of even natural number is 2, 4, 6, 8, ..., then nth term of the sequence is a_n given by $a_n = 2n$, where $n \in \mathrm{N}$.

   Reason (R) : If the sequence of odd natural numbers is 1, 3, 5, 7, ..., then nth term of the sequence is given by $a_n = 2n - 1$, where $n \in \mathrm{N}$.

   (i) Both assertion and reason are true and reason is the correct explanation of assertion.

(ii) Both assertion and reason are true but reason is not the correct explanation of assertion.

(iii) Assertion is true but reason is false.

(iv) Assertion is false but reason is true.

2. In each of the following questions, a statement of Assertion is given followed by a corresponding statement of Reason just below it. Of the statements, mark the correct answer as.

Assertion (A) : The fourth term of a GP is the square of its second term and the first term is −3, then its 7th term is equal to 2187.

Reason (R) : Sum of first 10 terms of the AP 6, 8, 10, ... is equal to 150.

(i) Both assertion and reason are true and reason is the correct explanation of assertion.

(ii) Both assertion and reason are true but reason is not the correct explanation of assertion.

(iii) Assertion is true but reason is false.

(iv) Assertion is false but reason is true.

Answer Key:

MCQ

1. (a) 0

2. (c) 3

3. (c) $a \times r^{n-1}$

4. (d) 1002001

5. (b) 1

6. (b) 1, 2, 4, 8

7. (c) $2 + \sqrt{3}$

8. (c) 402

9. (d) $1/2 - 1/\sqrt{2}$

10. (d) in H.P.

Short Answer:

1. Here a = 27 and d= $T_2 - T_1 = 24 - 27 = -3$.

Let the n^{th} term of series be 0.

∴ $T_n = a + (n - 1)d$

⇒ $0 = 27 + (n - 1)(-3)$

⇒ $3n - 3 = 27$

⇒ $3n = 30$

$n = 10$ or 10^{th} term.

2. Given series 8,4, 0, … (1)

First term = a = 8 Common difference d = 4 – 8 = -4

$d = 0 - 4 = -4$

∵ The common difference is same the series is in A.P.

last term $l = -24$,

$l = a + (n - 1)d$,

$= -24 = 8 + (n - 1)(-4)$

⇒ $-24 - 8 = (n - 1)(-4)$

⇒ $-32 = (n - 1)(-4)$

⇒ $(n - 1) = \frac{32}{4}$

⇒ $n - 1 = 8$

⇒ $n = 8 + 1$

⇒ $n = 9$

∴ Number of terms n = 9

3. Let first term = a and common difference = d of A.P.

∴ 7^{th} term = a + 6d and 11^{th} term = a + 10 d.

∴ According to question,

$7(a + 6d) = 11 + (a + 10d)$

⇒ $7a + 42d = 11a + 110d$

⇒ $7a - 11a = 110d - 42d$

⇒ $-4a = 68d$

⇒ $a = -17d$

Hence 18^{th} term = a + 17d

$= -17d + 17d$ [∵ $a = -17d$]

= 0.

4. Let the first term = a and common difference = d.

$T_n = a+(n-1)d$

$T_{m+n} = a + (m + n - 1)d$ … (1)

$T_{m-n} = a + (m - n - 1)d$ … (2)

$T_m = a + (m - 1)d$ … (3)

$T_{m+n} + T_{m-n} = a + (m + n - 1)d + a(m - n - 1)d$

$= 2a + (m + n - 1 + m - n - 1)d$

$= 2a + (2m - 2)d$

$= 2a + 2(m - 1)d$

$= 2[a+(m - 1)d]$

$T_{m+n} + T_{m-n} = 2T_m$.

5. Let three Arithmetic mean be A_1, A_2, A_3,

then 3, A_1, A_2, A_3, 19 are in A.P.

$\therefore$ 3 = a, 19 = T_5, let common difference = d

$T_5 = a + 4d$

$T_5 = a + 4d$

$\Rightarrow 19 = 3 + 4d$

$\Rightarrow 16 = 4d$

$\Rightarrow d = 4$

Hence $A_1 = 3 + 4 = 7$, $A_2 = 7 + 4 = 11$, $A_3 = 11 + 4 = 15$.

6. – 8, A_1, A_2, 9 are in A.P.

$\therefore$ a = – 8, T_4 = 9, let common difference = d

$T_4 = a + 3d$

$\Rightarrow 9 = -8 + 3d$

$\Rightarrow 3d = 11$

$\Rightarrow d = \frac{17}{3}$

7. Given : a = Rs. 100, d= Rs. 5, n = 30,

Tn = a + (n – 1)d

Amount of 30^{th} instalment

J30 = 100 + (30 – 1) x 5

= 100 + 29 × 5 = 100 + 145

= Rs. 245.

Hence the 30^{th} instalment = Rs. 245.

8.

Given : $a = \sqrt{3}, r = \frac{3}{\sqrt{3}} = \sqrt{3}$,

Let n^{th} term = 729

$\therefore \quad T_n = ar^{n-1}$

$\Rightarrow \quad 729 = \sqrt{3} \times (\sqrt{3})^{n-1}$

$\Rightarrow \quad 729 = (3)^{\frac{1}{2}+\frac{n-1}{2}}$

$\Rightarrow \quad 729 = 3^{\frac{1+n-1}{2}} \quad \Rightarrow \quad 3^6 = 3^{\frac{n}{2}}$

$\Rightarrow \quad \frac{n}{2} = 6 \quad \Rightarrow \quad n = 12$

Hence 12^{th} term = 729.

9.

Given : a = 3, r = $\frac{9}{3}$ = 3, S_n = 120,

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$\Rightarrow \frac{3(3^n - 1)}{3 - 1} = 120$$

$$\Rightarrow 3^n - 1 = \frac{120 \times 2}{3}$$

$\Rightarrow 3^n - 1 = 80$

$\Rightarrow 3^n = 81$

$\Rightarrow 3^n = 3^4$

$\Rightarrow$ n = 4

10. Let the roots of equation be α and β

A.M. of roots = $\frac{\alpha+\beta}{2} = 8$

$\Rightarrow \alpha + \beta = 16$

G.M. of roots = $\sqrt{\alpha\beta} = 5$

$\Rightarrow \alpha\beta = 25$

If α, β are the roots of equation

then, $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$\Rightarrow x^2 - 16x + 25 = 0.$

Long Answer:

1. a = 150, d = -4

$$S_n = \frac{n}{2}\left[2 \times 150 + (n-1)(-4)\right]$$

If total works who would have worked all n days 150(n-8)

$$\text{A T Q } \frac{n}{2}\left[300 + (n-1)(-4)\right] = 150(n-8)$$

$$n = 25$$

2.

Sn = 11 + 103 + 1005 + ---- + n terms

Sn = (10+1) + (102 + 3) + (103 + 5) + ----- + 10n+(2n−1)

$$S_n = \frac{10(10^n - 1)}{10 - 1} + \frac{n}{2}(\not{1} + 2n - \not{1})$$

$$= \frac{10}{9}(10^n - 1) + n^2$$

3.

$$\frac{\frac{a+b}{2}}{\sqrt{ab}} = \frac{m}{n}$$

$$\frac{a+b}{2\sqrt{ab}} = \frac{m}{n}$$

by C and D

$$\frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{m+n}{m-n}$$

$$\frac{\left(\sqrt{a}+\sqrt{b}\right)^2}{\left(\sqrt{a}-\sqrt{b}\right)^2} = \frac{m+n}{m-n}$$

$$\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{\sqrt{m+n}}{\sqrt{m-n}}$$

by C and D

$$\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{m+n}+\sqrt{m-n}}{\sqrt{m+n}-\sqrt{m-n}}$$

Sq both side

$$\frac{a}{b}=\frac{m+\not{n}+m-\not{n}+2\sqrt{m^2-n^2}}{m+\not{n}+m-\not{n}-2\sqrt{m^2-n^2}}$$

$$\frac{a}{b}=\frac{m+\sqrt{m^2-n^2}}{m-\sqrt{m^2-n^2}}$$

4.

$1, A_1, A_2, A_3, ---A_m, 31$ are in AP

a = 1

$a_n = 31$

$a_{m+2} = 314$

$a_n = a+(n-1)d$

$31 = a+(m+2-1)d$

$$d=\frac{30}{m+1}$$

$$\frac{A7}{A_{m-1}}=\frac{5}{9} \quad \text{(Given)}$$

$$\frac{1+7\left(\frac{30}{m+1}\right)}{1+(m-1)\left(\frac{30}{m+1}\right)}=\frac{5}{9}$$

$m = 1$

5.

$a+b=6\sqrt{ab}$

$$\frac{a+b}{2\sqrt{ab}}=\frac{3}{1}$$

by C and D

$$\frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}}=\frac{3+1}{3-1}$$

$$\frac{\left(\sqrt{a}+\sqrt{b}\right)^2}{\left(\sqrt{a}-\sqrt{b}\right)^2}=\frac{2}{1}$$

$$\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}=\frac{\sqrt{2}}{1}$$

again by C and D

$$\frac{\sqrt{a}+\sqrt{\not{b}}+\sqrt{a}-\sqrt{\not{b}}}{\sqrt{\not{a}}+\sqrt{b}-\sqrt{\not{a}}-\sqrt{b}}=\frac{\sqrt{2}+1}{\sqrt{2}-1}$$

$$\frac{2\sqrt{a}}{2\sqrt{b}} = \frac{\sqrt{2}+1}{\sqrt{2}-1}$$

$$\frac{a}{b} = \frac{\left(\sqrt{2}+1\right)^2}{\left(\sqrt{2}-1\right)^2} \text{ (on squaring both side)}$$

$$\frac{a}{b} = \frac{2+1+2\sqrt{2}}{2+1-2\sqrt{2}}$$

$$\frac{a}{b} = \frac{3+2\sqrt{2}}{3-2\sqrt{2}}$$

$$a : b = \left(3+2\sqrt{2}\right) : \left(3-2\sqrt{2}\right)$$

Assertion Reason Answer:

1. (ii) Both assertion and reason are true but reason is not the correct explanation of assertion.
2. (iv) Assertion is false but reason is true.